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# Measuring Bulk Properties of Sound-Absorbing Materials using the Two-Source Method

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## ABSTRACT

The two-source method was used to measure the bulk properties (complex characteristic impedance and complex wavenumber) of sound-absorbing materials, and results were compared to those obtained with the more commonly used two-cavity method. The results indicated that the two-source method is superior to the two-cavity method for materials having low absorption. Several applications using bulk properties are then presented. These include: (1) predicting the absorptive properties of an arbitrary thickness absorbing material or (2) layered material and (3) using bulk properties for a multi-domain boundary element analysis.

## INTRODUCTION

Sound-absorbing materials are commonly used in industry to reduce noise. Normally, the sound absorption coefficient and surface impedance are used to characterize sound-absorbing materials and are sufficient for many applications. These properties can be measured using the two-microphone method described in ASTM E1050-98 [1] provided that a sample of appropriate thickness is on hand. However, knowing the bulk properties provides the necessary information to predict absorptive properties of materials of arbitrary thickness, and also of layered materials. Furthermore, bulk properties are more suitable in computational models of absorptive components like seats or thick sections of sound-absorbing materials. This paper will focus on using the two-source method to measure absorptive properties and then consider a few applications.

A homogeneous porous material may be described in terms of its bulk properties. These properties consist of the complex characteristic impedance and the complex wavenumber. One way to obtain the bulk properties is to use a empirical or semi-empirical prediction based on a regression analysis of measured data [2]. However, the accuracy depends on how similar the material in question is to the material that was used to develop the equations. Another way to obtain the bulk properties is to measure them experimentally. Several methods [3-7] have been used. Most relate the bulk properties to the measured surface impedance. Two such methods are the two-thickness method [4] and the two-cavity method [5]. The two-cavity method is the easier of the two since only one sample is needed. It should be noted that an improved two-cavity method [6] was used with an improved measurement technique [1,8].

A second way to measure the bulk properties, which will be introduced in this paper, is a two-source method. This method takes advantage of the transfer matrix method [9]. In short, an acoustical element like a muffler or a section of absorptive material can be described by its so-called four-pole parameters assuming plane wave propagation. These four-pole parameters can be calculated from microphone measurements. The experimental setup involves the use of two sources and measurements at four microphone locations. It should be noted that Song and Bolton [7] proposed a similar method using a single source making use of the reciprocity and symmetry of the homogeneous and isotropic material.

Once the bulk properties are obtained, (1) the absorption coefficient and surface impedance with arbitrary thickness or with an arbitrary air space depth can be predicted [6,10], (2) the absorptive properties of layered materials can be calculated using the transfer matrix method, and (3) a numerical analysis with bulk-reacting materials can be performed [10,11].

In this paper, the two-source method is compared with the two-cavity method for measuring the bulk properties of an absorptive material. The theory for each method is presented first, followed by examples that show the differences between the two methods. Results from the two-source method are then used in several applications.

## MEASUREMENT OF BULK PROPERTIES OF ABSORBING MATERIAL

TWO-CAVITY METHOD [6]

A schematic showing the experimental setup for the two-cavity method is shown in Figure 1. Two separate tests are conducted in accordance with ASTM E1050-98 [1], and the length of the cavity behind the sample is changed for each test. The surface impedances $z_1$ and $z_1$' are measured with two air cavities having lengths $L$ and $L$', respectively. The length of the cavity can be changed by moving a piston along the impedance tube. Then a complex wavenumber $k$' and a complex characteristic impedance $z_c$ can be derived by plane wave theory and expressed as

$$z_c = \pm\sqrt{\frac{z_1 z_1'(z_2 - z_2') - z_2 z_2'(z_1 - z_1')}{(z_2 - z_2') - (z_1 - z_1')}} \quad (1)$$

and

$$k' = \left(\frac{1}{2jd}\right)\ln\left(\frac{(z_1 + z_c)(z_2 - z_c)}{(z_1 - z_c)(z_2 + z_c)}\right) \quad (2)$$

where

$$z_2 = -jz_0\cot(kL),\ \text{and}\ \ z_2' = -jz_0\cot(kL'), \quad (3)$$

The sign in Equation (1) is selected so as to let the real part of $z_c$ be positive. $z_1$ and $z_1'$ are the measured impedances for air cavity lengths of $L$ and $L$', respectively, $z_2$ and $z_2'$ are the impedances of the air cavities of lengths $L$ and $L$', respectively, $d$ is the thickness of the sample, $z_0$ is the characteristic impedance of air, and $k$ is the wavenumber.

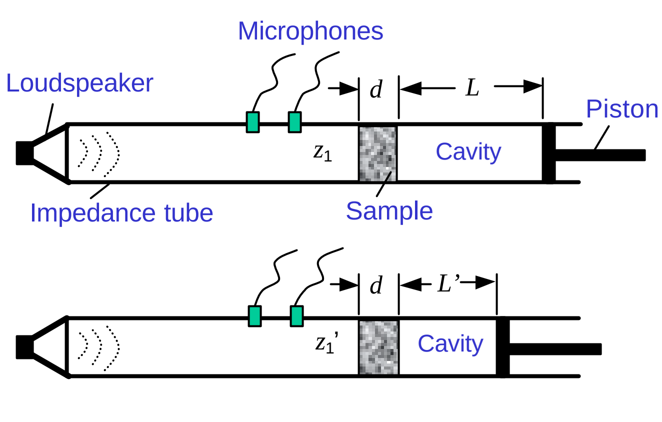


**Figure 1** Setup of the two-cavity method

For the two measurement situations, the difference between $L$ and $L$' determines the upper cut-off frequency $f_u$ of the measurement

$$f_u = \frac{c}{2(L - L')} \quad (4)$$

Once the complex characteristic impedance and the complex wavenumber are obtained, the complex speed of sound $c_c$ and complex density $\rho_c$ for the sound-absorbing material are found from

$$c_c = \omega / k' \quad \text{and} \quad \rho_c = z_c / c_c\,, \quad (5)$$

where $\omega$ is the angular frequency.

TWO-SOURCE METHOD [9]

The two-source method is based on the transfer matrix approach. An acoustical element can be modeled by its four-pole parameters which relate the sound pressure and particle velocity on each side of the absorbing material (illustrated in Figure 2). The transfer matrix can be expressed as

$$\begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} p_2 \\ v_2 \end{bmatrix}, \quad (6)$$

where $p_1$ and $p_2$ are the sound pressure amplitudes; $v_1$ and $v_2$ are the particle velocity amplitudes, and $A$, $B$, $C$ and $D$ are the four-pole parameters of the system.

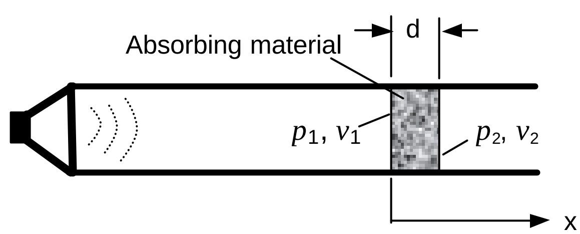


**Figure 2** The four-poles

When using the two-source method, the sound source should be placed in each of two configurations ($a$ and $b$) as shown in Figure 3. The experimental setup can be thought of as having three distinct elements. An element is defined between each microphone location. Thus, the tube is made up of elements 1-2, 2-3, and 3-4. Element 2-3 can be further divided into three sub-elements: sub-elements 2-5, 5-6, and 6-3.

By solving the equations for different elements, the four-pole parameters for element 2-3 can be written as

$$A_{23} = \frac{\Delta_{34}(H_{32a}H_{34a} - H_{32b}H_{34a}) + D_{34}(H_{32b} - H_{32a})}{\Delta_{34}(H_{34b} - H_{34a})} \quad (7)$$

$$B_{23} = \frac{B_{34}(H_{32a} - H_{32b})}{\Delta_{34}(H_{34b} - H_{34a})} \quad (8)$$

$$C_{23} = \frac{(H_{31a} - A_{12}H_{32a})(\Delta_{34}H_{34b} - D_{34}) - (H_{31b} - A_{12}H_{32b})(\Delta_{34}H_{34a} - D_{34})}{B_{12}\Delta_{34}(H_{34b} - H_{34a})} \quad (9)$$

$$D_{23} = \frac{(H_{31a} - H_{31b}) + A_{12}(H_{32b} - H_{32a})}{B_{12}\Delta_{34}(H_{34b} - H_{34a})} B_{34}\,, \quad (10)$$

where the subscripts $a$ and $b$ refer to Configurations $a$ and $b$ in Figure 3, respectively; $H_{ij} = p_j / p_i$; $D_{12} = A_{12}D_{12} - B_{12}C_{12}$, and $D_{34} = A_{34}D_{34} - B_{34}C_{34}$. The four poles for elements 1-2 and 3-4 are

$$\begin{bmatrix} A_{12} & B_{12} \\ C_{12} & D_{12} \end{bmatrix} = \begin{bmatrix} \cos kl_{12} & j\rho c \sin kl_{12} \\ j \sin kl_{12}/(\rho c) & \cos kl_{12} \end{bmatrix}, \quad \Delta_{12} = 1 \quad (11)$$

and

$$\begin{bmatrix} A_{34} & B_{34} \\ C_{34} & D_{34} \end{bmatrix} = \begin{bmatrix} \cos kl_{34} & j\rho c \sin kl_{34} \\ j \sin kl_{34}/(\rho c) & \cos kl_{34} \end{bmatrix}, \quad \Delta_{34} = 1 \quad (12)$$

respectively. In Equations (11)-(12), $l_{12}$ and $l_{34}$ are the microphone spacings for elements 1-2 and 3-4, respectively.

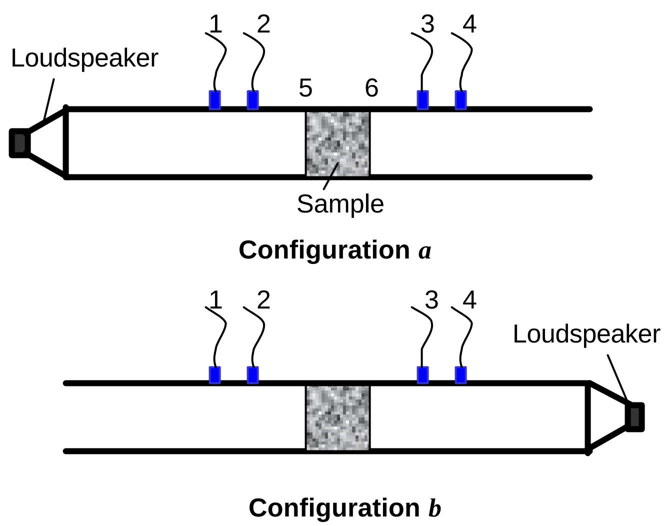


**Figure 3** Setup of the two-source method

It should be noted that the two-source method can be implemented using only two microphones. One can obtain all necessary transfer functions $H_{ij}$ by moving one microphone and using the other microphone as a reference. It is also possible to reverse the sample, which may be easier than moving the sound source. Normally, random excitation is used.

Recall that element 2-3 can be expressed in terms of its sub-elements 2-5, 5-6, and 6-3. The four-pole matrix for element 2-3 can then be expressed as

$$\begin{bmatrix} A_{23} & B_{23} \\ C_{23} & D_{23} \end{bmatrix} = \begin{bmatrix} A_{25} & B_{25} \\ C_{25} & D_{25} \end{bmatrix} \begin{bmatrix} A_{56} & B_{56} \\ C_{56} & D_{56} \end{bmatrix} \begin{bmatrix} A_{63} & B_{63} \\ C_{63} & D_{63} \end{bmatrix}, \quad (13)$$

and the four-pole parameters of the absorbing material are obtained by

$$\begin{bmatrix} A_{56} & B_{56} \\ C_{56} & D_{56} \end{bmatrix} = \begin{bmatrix} A_{25} & B_{25} \\ C_{25} & D_{25} \end{bmatrix}^{-1} \begin{bmatrix} A_{23} & B_{23} \\ C_{23} & D_{23} \end{bmatrix} \begin{bmatrix} A_{63} & B_{63} \\ C_{63} & D_{63} \end{bmatrix}^{-1}. \quad (14)$$

The four-pole parameters of the straight tube elements 2-5 and 6-3 can be obtained by formulae like those shown in Equations (11) or (12).

Since the four-pole parameters of the sound-absorbing material can be expressed as

$$\begin{bmatrix} A_{56} & B_{56} \\ C_{56} & D_{56} \end{bmatrix} = \begin{bmatrix} \cos(k'd) & jz_c \sin(k'd) \\ j \sin(k'd)/z_c & \cos(k'd) \end{bmatrix}. \quad (15)$$

the bulk properties can be found from the four-pole parameters. Specifically, the complex wavenumber and complex characteristic impedance can be calculated by

$$k' = \frac{1}{d}\cos^{-1} A_{56} \quad (16)$$

and

$$z_c = \sqrt{B_{56}/C_{56}} \quad (17)$$

respectively.

Figures 4 and 5 shows the measured $k'$ and $z_c$, respectively, for a polyester absorbing material using the two methods. One can see that both methods compare closely for this material which has high absorption.

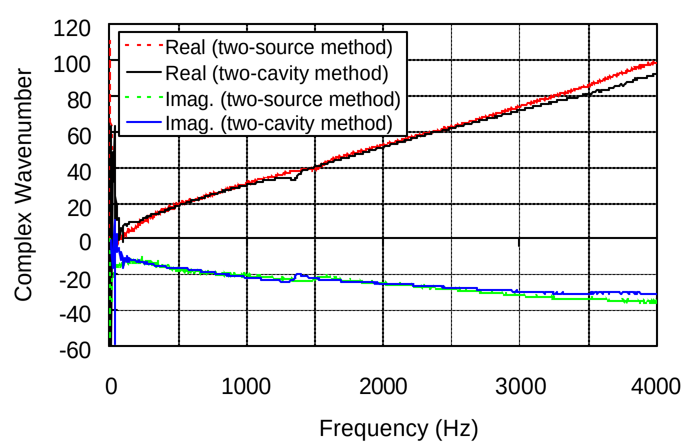


**Figure 4** Measured complex wavenumber of the polyester material

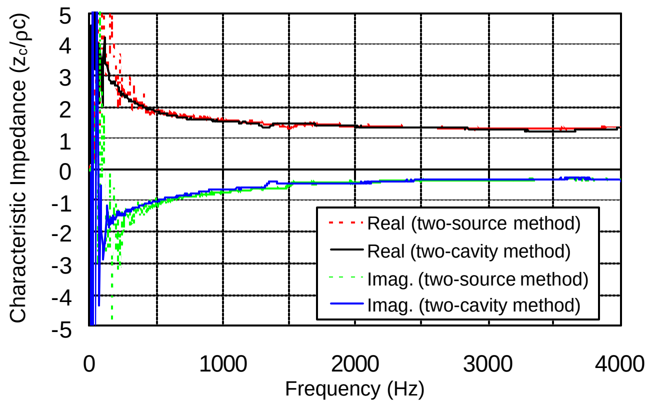


**Figure 5** Measured complex characteristic impedance of the polyester material

Figure 6 shows the measured complex wavenumber for a material having low absorption (absorption coefficient is below 0.4 at 4000 Hz). The figure indicates that more reliable results are obtained using the two-source method. A low absorption suggests two possibilities that can lead to problems with the two-cavity method. (A) If an absorbing material reflects most of the sound (i.e. a "harder surface"), changing the tube length will have little effect on the surface impedance measurements. (B) Alternatively, if the absorbing material is very thin, the cavity impedances are approximately equal to the surface impedances. This problem (B) can be solved by just adding additional thickness to the sample so that the impedance difference between $z_1$ and $z_2$ is more appreciable. However, problem (A) cannot be solved as easily using the two-cavity method. The material shown in Figure 6 had properties consistent with problem (A).

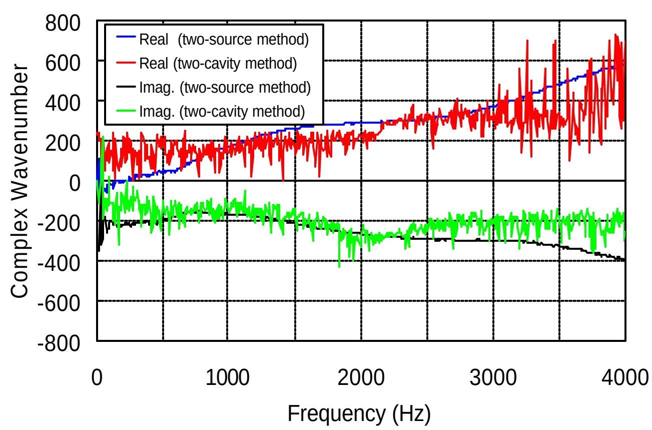


**Figure 6** Measured complex wavenumber of a low-absorbing material

VERIFICATION

The accuracy of the measured $k'$ and $z_c$ can be verified by comparing the measured absorption coefficient or surface impedance to that calculated using $k'$ and $z_c$ for no air gap behind the sample, i.e., $L = 0$. The surface impedance $z_{in}$ for a thickness $d$ of material in front of a rigid surface is

$$z_{in} / \rho c = -j z_c \cot(k'd) / \rho c = r' + jx' \qquad (18)$$

and the absorption coefficient $\alpha(\phi)$ for any angle of incidence $\phi$ can be obtained from

$$\alpha(\phi) = \frac{4r'\cos\phi}{(1 + r'\cos\phi)^2 + (x'\cos\phi)^2} \qquad (19)$$

Figure 7 shows the comparison of the measured and calculated absorption coefficient for a sample of 1 in. thick polyester for normal incidence ( $\phi$=0 ). One can see that the calculated and measured results match very well, suggesting that the measured $k'$ and $z_c$ are reliable.

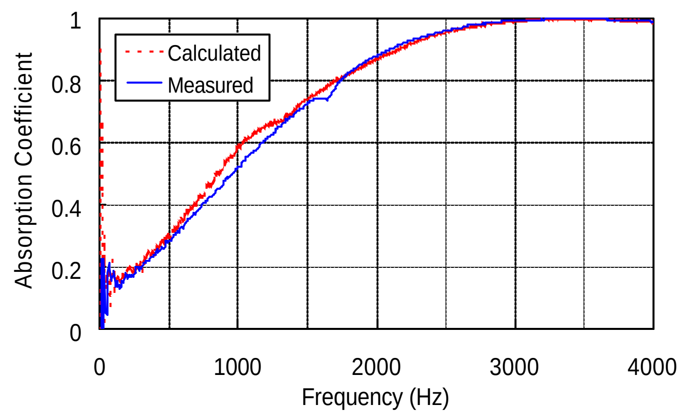


**Figure 7** Absorption coefficient comparison, calculated vs. measured for $\phi$=0

## APPLICATIONS

PREDICTION OF ABSORPTION PROPERTIES FOR ARBITRARY THICKNESS ABSORBING MATERIAL

Once the bulk properties are measured by the two-cavity or the two-source method, the absorptive properties of an arbitrary thickness material can be evaluated by Equations (18)-(19). Figure 8 shows the absorption coefficient comparison for the same material as that used in Figure 7 except with the thickness doubled. Good agreement was achieved for the absorption calculation.

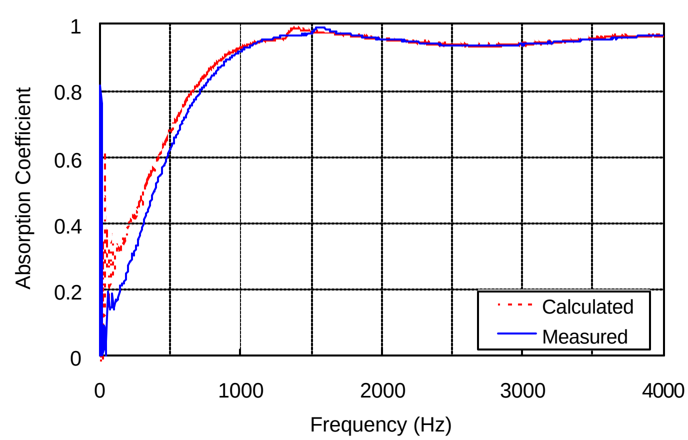


**Figure 8** Absorption coefficient comparison, calculated vs. measured for $\phi$=0

TRANSFER MATRIX METHOD FOR LAYERED MATERIALS

For a layered material, like that shown in Figure 9, sound pressure and the particle velocity at the contact surfaces of the layered materials can be expressed by

$$\begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = \left[T_{total}\right] \begin{bmatrix} p_{n+1} \\ v_{n+1} \end{bmatrix} , \qquad (20)$$

where $\left[T_{total}\right]$ is the total acoustic transfer matrix from Layer 1 to Layer $n$, obtained by multiplying the transfer matrices of the individual layers, $T_1$, $T_2$, …, $T_n$, i.e.,

$$\left[T_{total}\right] = \left[T_1\right]\left[T_2\right]...\left[T_n\right] = \begin{bmatrix} A_T & B_T \\ C_T & D_T \end{bmatrix} . \qquad (21)$$

where $A_T$, $B_T$, $C_T$ and $D_T$ are the overall four-pole parameters of Layer 1 to Layer n. It is evident that the four-pole parameters for each layer can be obtained by the two-source method directly using Equation (14). As mentioned previously, the two-source method could be used to obtain the complex density and complex speed of sound. Once the complex density and complex speed of sound are obtained, Equation (15) could be used to define the four-pole parameters for each layer.

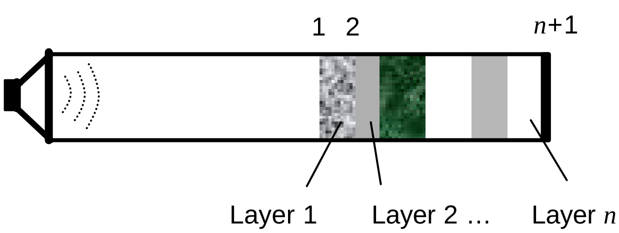


**Figure 9** A layered material

For a rigid surface at location $n$+1, the reflection coefficient $R$ for $\phi$=0 is [7]

$$R = \frac{A_T - rcC_T}{A_T + rcC_T} . \qquad (22)$$

It follows that the normalized surface impedance can be obtained from

$$z_{in} / \rho c = \frac{1+R}{1-R} = \frac{A_T}{C_T \cdot \rho c} , \qquad (23)$$

and the absorption coefficient from

$$\alpha = 1 - \left|R\right|^2 . \qquad (24)$$

Figures 10 and 11 show comparisons between the calculated and measured surface impedance and absorption coefficient for a two-layered foam with an air gap behind. Again, the calculated results compared well with the measured results.

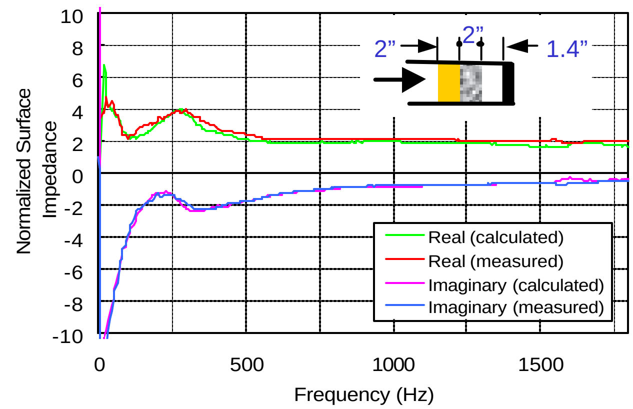


**Figure 10** Normalized surface impedance calculated vs. measured

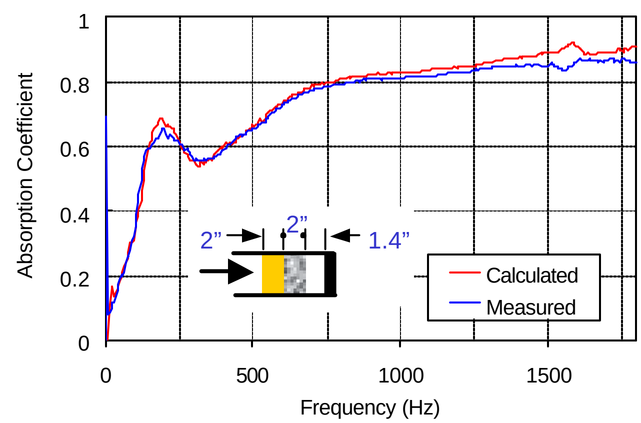


**Figure 11** Absorption coefficient comparison calculated vs. measured for $\phi$=0

MULTI-DOMAIN BEM ANALYSIS [10, 11]

For packed or hybrid mufflers, there are at least two different acoustic media involved: air and a bulk-reacting sound absorbing material. In a BEM analysis, a sound-absorbing material can be modeled as either a locally-reacting or a bulk-reacting material. In the local-reacting approach, the surface impedance is used as a boundary condition. In the bulk-reacting approach, a multi-domain or direct-mixed body BEM analysis should be conducted, using bulk-reacting properties to model the absorption.

The packed expansion chamber shown in Figure 12 was used in Reference 11. The results indicate that when the thickness of the absorbing material is $h$ = 0.5 in., both approaches produce a very similar result, but when the thickness of the absorbing material is increased to $h$ = 1 in., the transmission loss (TL) predicted using the bulk-reacting approach is better than that obtained by the local-reacting approach, as shown in Figure 13.

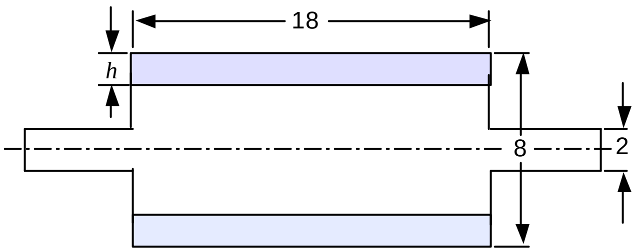


**Figure 12** A packed expansion chamber (dimensions in inches)

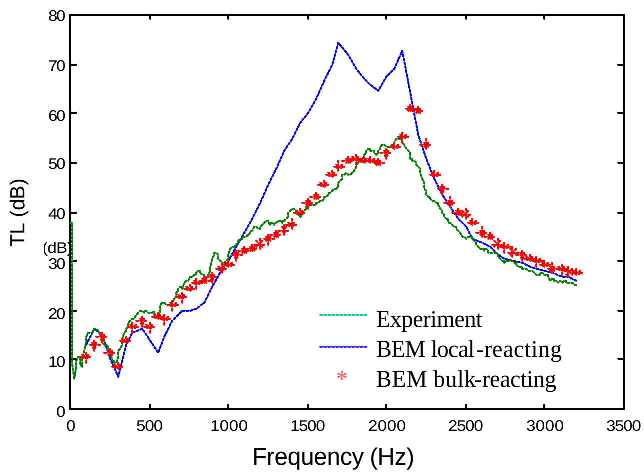


**Figure 13** Transmission loss of the expansion chamber muffler

## CONCLUSIONS

The measurement of the bulk properties of sound-absorbing materials has been discussed in this paper. The two-cavity and the two-source methods were compared, and the results indicate that both the two-cavity method and the two-source method can be utilized to measure the bulk properties of an absorbing material. Though the two-cavity method is easy to employ, the two-source method works better than the two-cavity method for materials having low absorption.

Applications showing the use of bulk properties were also presented. The results show that the bulk properties can be used to predict the absorptive properties of an arbitrary thickness material and also may be used for layered materials. Additionally, the advantage of using bulk properties in certain BEM analyses was also illustrated.

## ACKNOWLEDGMENTS

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